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CONSENSUS FORMATION IN MONETARY POLICY COMMITTEES

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Consensus Formation in Monetary Policy Committees*

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Abstract

Building on Blinder and Wyplosz (2005) this paper presents a formal mechanism which potentially explains how *autocratically collegiate*, *genuinely collegiate* and *individualistic* monetary policy committees (MPCs) are able to reach a consensus. Drawing on the theory of Markov chains, I adopt a bounded-rational approach, and demonstrate how individuals are able to forge agreement, even when interest rate preferences are initially diverse. I show how consensus is reached when (*i*) career concerns are present and (*ii*) when members hold different opinions about the usefulness of others' information. An overriding conclusion which emerges is that it is possible to populate MPCs with people who hold very different views about the economy and *still* reach an agreement. Further, although MPCs should be populated by people who are willing to listen to the opinions of others, the *degree* to which members are willing to listen to each other has ramifications for the type of decision which is reached.

Keywords: Monetary Policy Committee, consensus formation, bounded-rationality.

Contents

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1 Introduction

In this paper I develop a *boundedly-rational* model of how monetary policy committees, as characterised by Blinder and Wyplosz (2005), are able to reach decisions on the interest-rate. I develop a theoretical framework which draws upon Morris DeGroot's (1974) characterization of consensus formation in groups and DeMarzo, Vayanos and Zweibel's (2003) model of belief convergence in social networks. Given the role of consensus formation in real-world MPCs,[1] both models, which share common features, are of relevance to the modelling of MPC decisions. Implicit to each approach is the use of Markov chains as a heuristic by which agents of a group or network update their beliefs in successive periods, a procedure hereafter referred to as *Markovian Updating* (MU). It provides a dynamic mechanism by which MPC members align their views through the deliberation process. Within this framework, it is demonstrated how members of an MPC are able to reach agreement even when (*i*) not all members listen to each other in the course of deliberations and (*ii*) views of members are initially diverse. The extent to which members of a group listen to each

[1] Fry *et al* (2000).

other is modelled using a *transition matrix*, the configuration of which determines whether members are able to agree with each other, and if so, the nature of any such agreement. To compliment the analysis, I also use *graph-theoretic representation* to depict how members of a monetary policy committee are connected to each other.

The model I develop potentially explains how members of the United States FOMC, ECB *Governing Council* and Bank of England MPC are able to reach a agreement on the interest-rate. I suggest, for example, that it is capable of explaining the stylised facts of Bank of England MPC member voting behaviour. In particular, the voting record of the BoEMPC suggests that on a meeting by meeting basis, policy preferences of outsiders exhibit significantly more *heterogeneity* than those of insiders. To quote Edmonds (1999) insiders behave as if they are a "cohesive homogeneous group."[2] Outsiders, by contrast constitute a more disparate grouping. The model potentially explains this behaviour. It is also applicable to FOMC voting behaviour. Recent studies of FOMC decision making have characterised its Chairman as a monetary policy 'dictator'. Chappell *et al* (2002) report a disproportionately *large* influence of the Chairman in relation to final policy decisions when compared with other FOMC members. Again, the general model developed herein plausibly reproduces this phenomenon.

The plan of the paper is as follows. I initially proceed by presenting some 'stylised facts' of monetary policy committee voting. When presenting these facts, I draw on Blinder and Wyplosz's (2005) classification of real-world monetary policy committees into three distinct varieties - those which are (*i*) *autocratically-collegial*, (*ii*) *genuinely-collegial* and (*iii*) *individualistic*. It is noted that this classification is particularly useful when rationalising the results of model simulations presented towards the end of the paper. I then proceed to outline the models of DeGroot (henceforth DG) and DeMarzo, Vayanos and Zweibel (henceforth DVZ). In much that same way DVZ suppose that newspapers sway readers toward their views over time - even when the political affiliation of a newspaper is common knowledge - some members of a monetary policy committee sway other members to their views, even when all member's views are commonly known. A framework is provided which is able to account for why some MPC members listen to some of their colleagues, but not others. I rationalise this in terms of amongst other factors, the career concerns of MPC members and the precision of members' information. Further, I account for why members of an MPC might hold different views regarding the appropriate policy stance. This, I suggest is attributable to the theoretical leanings of MPC members and their perception of how the economy works. All of these claims are supported by empirical evidence, whether in the form of MPC voting records or comments or statements made by members of MPCs past and present. I now turn to the empirics.

2 Some Stylised Facts of MPC Voting

I begin by presenting some stylised facts of member voting behaviour for the United States FOMC, the ECB *Governing Council* and the Bank of England MPC. Blinder and Wyplosz's (2005) characterisation of real-world monetary policy committees into three distinct types is integral to the analysis, and it is notable that according to their classification, none of the three committees discussed here are of the same type. Most pertinently, the classification method of Blinder and

[2] Edmonds (1999), p.3.

Wyplosz - hereafter BW - identifies the characterising structures of each committee, in addition to rationalising the different patterns of voting behaviour corresponding to each one. It explains, for instance, why different monetary policy committees experience different levels of dissent.

It is noted however that the first two types of committee I define fall in a more general class of *collegial committees*, according to which the decision reached by an MPC is supported by *all* members: the policy decision is seen as embodying the collective wisdom of the committee, and its members hold that any differences of opinion must be second to the common good - otherwise the authority of the committee is diminished.[3] This is true for both committees reach decision by taking a formal vote and reaching a consensus. For those committees where a formal vote *is* taken, it is typified by a unanimous decision, or near unanimity. *Dissenting* votes are thus considered unusual. So-called *autocratically-collegial*, *genuinely-collegial* and *individualistic* monetary policy committees are now defined and related to their respective real-world counterparts, the US FOMC, ECB Governing Council and Bank of England MPC.[4]

2.1 Autocratically-Collegial MPCs

In an *autocratically-collegial committee*, the chairman is a virtual monetary policy 'dictator'. The interest-rate decision is effectively the Chairman's choice. He may make a decision prior to the meeting, and merely notify his colleagues at its outset. Alternatively, he might take on board the views of other committee members *during* the meeting, then announce his decision and expect everyone to close ranks. BW class the US FOMC under Alan Greenspan as an *autocratically collegial committee*, an assertion which is borne out by the recent FOMC voting record. Between $2000 - 2004$, of 43 FOMC meetings and 473 votes cast excluding those of the chairman, only six were classed as dissenting.[5] Further work by Chappell *et al* (2002) reconfirms this result. They report that even though the FOMC places a very high value on reaching a consensus, the Chairman exercises 40-50% of the voting weight in committee decisions. In similar vain, Maisel (1973) argues that although the Chairman may be influenced by other FOMC members, *any* policy preferred by him is likely to be adopted.

2.2 Genuinely-Collegial MPCs

Members on a *genuinely-collegial committee* may openly disagree strongly about the best policy stance in the course of MPC deliberations but in the end compromise on a committee decision.

[3] Consider for example the comments of Chairman Burns from September 20, 1977 when the FOMC was badly split by seven votes to five in favour of the first policy directive:

> "Well let's stop and deliberate it. I think that would be a very unfortunate vote. *It would mean that this would excite a great deal of discussion that would not bring honour or credit to the Committee and therefore I think we must seek to accommodate one another.* I don't think that our differences are very large. Let's try again. Does anyone have a proposal to make, one of the dissenters?" (*Emphasis added.* Cited from Chappell *et al* (2002), pp5-6.)

[4] BW also suggest there exists a natural ordering to their classification of committee types in terms of *closeness* to the unitary decision maker of economic theory. As one moves further down the ranking, so too does the power and influence of any single committee member, such as the Chairman. In terms of closeness to a single policy maker - consider the atypical institutional arrangement at the Reserve Bank of New Zealand where the decision on the interest rate rests solely with the Governor - the order of closeness is *autocratically-collegial* MPC, *genuinely-collegial* MPC and finally an *individualistic MPC*.

[5] As the Chairman tables the motion, he is assumed not to dissent.

Once a compromise is reached, and the decision is announced, all committee members present a united front in public, ensuring that any disagreements are left in the board room. Essentially, each member effectively assumes ownership of the decision. The ECB Governing Council is an example of such a committee. Although minutes of ECB Governing Council meetings are not published, it is well known that *no* formal vote is taken by its members - rather, all members reach a consensus,[6] with all members' opinions reportedly converging on a single interest-rate.[7] This assertion has a basis in numerous answers provided by the President of the ECB, Wim Duisenberg, to questions fielded at the routine ECB press conferences which follow monetary policy decisions made by the Governing Council. Remarks made on February 3^{rd} 2000 reflect this:

> "First, there was no formal vote. Again....it was a *consensus* decision." (*emphasis added*)

Similar comments were made on June 8^{th} 2000:

> "We had an intensive discussion, a prolonged discussion, which was very useful, and, in the end, resulted in a *consensus* on what we had to do." (*emphasis added*)

2.3 Individualistic MPCs

On an *individualistic committee*, members not only openly disagree strongly about the best policy stance in the course of MPC deliberations, but actively cast a vote which reflects their position. Such a committee is assumed to make decisions through the application of SMV, with a unanimous decision being neither expected nor sought. The Bank of England MPC is the archetypal case of an *individualistic committee*. The rate of dissent is certainly higher on the Bank of England MPC than the FOMC. Meade and Sheets (2002) report that over the period 1978-2000 inclusive, only 198 out of 2403 votes cast by FOMC members were dissenting. This amounts to about 8% of all votes cast. Contrast this with the voting behaviour of Bank of England MPC members. For the first five years of the MPC, 106 out of 642 votes cast - approximately $16\frac{1}{2}\%$ - were dissenting. Crudely put, MPC members are twice as likely to dissent than FOMC members.[8] However, it is noted here that on an individualistic MPC, the prospect of a *majority* of members not being able to reach agreement is emphatically *not* an option: irrespective of any differences over the appropriate interest-rate for the economy, MPC members are compelled to reach a decision, albeit via a winning majority as opposed to unanimity. Clearly, failure to reach a decision in the form of no winning

[6] I thank Nick Vidalis and Marco Catenaro at the ECB for helpful discussions relating to this matter.

[7] This is in spite of ECB statutes stating that decisions taken by the GC on the short term interest rate are to be taken using the mechanism of simple majority rule.

[8] Comparison with other committees is also of interest. Nobuyuki Nakahara (2001), member of the Policy Board of the Bank of Japan, attributes differences in the dissent voting behaviour of members of the monetary policy committees of the Bank of England and Bank of Japan to *individual accountability*, attesting:

> "I heard that Dr. DeAnne Julius, a former member of the Monetary Policy Committee of the Bank of England, said that when members are not individually accountable, they lose the incentive to make public their position at the voting stage even if they had voiced opposing views during the debate, and that it will become easier for the majority, which would include the most influential individual, to carry the vote. To avoid this situation, the parliament holds individual hearings. Although the connection is not clear, since April 1998, deputy governors, though they are chosen from the staff of the BoE, are known to have cast eleven minority votes on eight occasions. As for the Bank of Japan, it was revealed at a recent parliamentary session that there had never been a division of views of the governor and two deputy governors." (emphasis added)

majority emerging would have damaging consequences for the credibility of monetary policy.[9] I now turn to the model.

3 The Model

Envisage a monetary policy committee of m members with responsibility for setting the interest-rate. Prior to the start of the meeting (and the deliberation process), each MPC member *weights* the opinions of other members, *including himself.* More formally, let, $\mathsf{p}_{j,k}$ denote the weight placed on member k's opinion by the j^{th} member. For each member the sum of weights equals one

$$\sum_{j=1}^{m} \mathsf{p}_{j,k} = 1 \tag{1}$$

where

$$0 \leq \mathsf{p}_{j,k} \leq 1 \qquad \forall\ j,k \in \{1,2,...m\} \tag{2}$$

This determines the elements of an $m \times m$ *transition* matrix, where each row corresponds to respective members' allocation of weights.

$$\mathsf{P} = \begin{bmatrix} \mathsf{p}_{1,1} & \mathsf{p}_{1,2} & \cdot & \cdot & \cdot & \mathsf{p}_{1,m-1} & \mathsf{p}_{1,m} \\ \mathsf{p}_{2,1} & \mathsf{p}_{2,2} & \cdot & \cdot & \cdot & \mathsf{p}_{2,m-1} & \mathsf{p}_{2,m} \\ \cdot & \cdot & \mathsf{p}_{3,3} & & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & & \mathsf{p}_{m-2,m-2} & \cdot & \cdot \\ \mathsf{p}_{m-1,1} & \mathsf{p}_{m-1,2} & \cdot & \cdot & \cdot & \mathsf{p}_{m-1,m-1} & \mathsf{p}_{m-1,m} \\ \mathsf{p}_{m,1} & \mathsf{p}_{m,1} & \cdot & \cdot & \cdot & \mathsf{p}_{m,m-1} & \mathsf{p}_{m,m} \end{bmatrix} \tag{3}$$

I call the configuration of P the *influence structure* of the group. P is informative because it indicates the extent to which all members of the group are *influenced* by each other. Specifically, I define $\mathsf{p}_{j,k}$, the weight placed on member k's opinion by the j^{th} member, as the *direct influence* of k on j. Accordingly, k is said to have no *direct influence* on j if $\mathsf{p}_{j,k} = 0$. However, if $\mathsf{p}_{j,k} = 0$, it is still possible for k to influence j. Even when member j is not *directly* influenced by k, k bears influence on j if k influences a member l who *directly influences* j. Call this the *indirect influence* of j on k.[10] Moreover, k still exerts influence on j if k influences a member l who *indirectly* influences j, and so on. This example illustrates that for some weighting allocations the nature of influence running to and from members may become highly complex, with the chain of influence amongst members being deeply intertwined. By way of a further definition if a member j neither *directly* nor *indirectly influences* member k, then that member is not *influenced* by k. Further, if any two members j and k do not *influence* each other, they they do not *communicate* with each other. More generally, in the same way that m individuals do not communicate with each other an analogous situation arises for *groups* of individuals. If there are two groups, J and K where no member of either group communicates with each other then group J does not communicate with group K. This logic extends to the case of $m > 2$ groups.

[9] It is inconceivable that a monetary policy committee would announce to the public that "our members are unable to reach a collective decision about the level of the interest-rate. Come back again tomorrow."

[10] We note here that k's influence on l may be either *direct* or *indirect* in nature.

3.1 Interest-Rate Preferences

Corresponding to P is a *belief* vector containing members' interest-rate preferences *prior* to the deliberation process. This vector contains the interest rates members would choose to set were they given *individual* responsibility for monetary policy. Denote the transpose of this vector as

$$\mathsf{I}^{[0]\prime} = [\mathsf{i}_{1,0}, ..., \mathsf{i}_{m,0}] \tag{4}$$

where numbers given in subscripts $_{M,N}$ correspond to the respective interest-rate preferences for members $M = \{1, ...m\}$ and the stage of the deliberation process $N = \{1, 2, ..., n\}$ which is also denoted in the square bracket $\mathsf{I}^{[N]}$. Members' revised views after the first period of discussions are calculated by pre-multiplying the vector $\mathsf{I}^{[0]}$ by P, yielding

$$\mathsf{I}^{[1]} = \mathsf{P}\mathsf{I}^{[0]} \tag{5}$$

The transpose of this vector is given by

$$\mathsf{I}^{[1]\prime} = [\mathsf{i}_{1,1}, ..., \mathsf{i}_{m,1}] \tag{6}$$

Consensus is reached by an *iterative* process. Following the first round of deliberations members' original interest-rate preferences will have changed from $\mathsf{i}_{1,0}..., \mathsf{i}_{m,0}$ to revised estimates given by $\mathsf{i}_{1,1}, ..., \mathsf{i}_{m,1}$. If a majority of members' revised rates have not converged to the same interest-rate in the first period, then the process of revision continues until it does. Revised opinions are calculated up to the n^{th} period as

$$\begin{aligned}
\mathsf{I}^{[1]} &= \mathsf{P}\mathsf{I}^{[0]} = \mathsf{P}^1\mathsf{I}^{[0]} \\
\mathsf{I}^{[2]} &= \mathsf{P}\mathsf{I}^{[1]} = \mathsf{P}^2\mathsf{I}^{[0]} \\
&. \\
&. \\
&. \\
\mathsf{I}^{[n]} &= \mathsf{P}\mathsf{I}^{[n-1]} = \mathsf{P}^n\mathsf{I}^{[0]}
\end{aligned} \tag{7}$$

where P^n is the matrix P raised to the n^{th} power, $n = 1, 2, ...n$. The mechanism by which members revise their opinion might best be best viewed as a *discrete iterative process* that only ceases when all members reach a consensus.[11] With each iteration, members revise their judgement of the interest-rate given their weighting of others' opinions.

3.2 Informational Criterion

An alternative criterion to allocating weights in the influence matrix is to assume that MPC members weight members according to informational criterion as opposed to perceived career concerns. I now envisage a setting where MPC members must estimate the appropriate interest-rate for the economy, i^*, in order to achieve some pre-determined policy objective, such as hitting an inflation target.[12] The approach taken follows elements of DVZ, particularly with respect to the structure of

[11] In the case of simple majority rule, it might only be necessary for half of MPC members to reach a consensus amongst themselves for a majority decision to be reached.

[12] Such as the institutional arrangements at the Bank of England.

the *influence matrix*, or to use their terminology, *listening matrix*. Members are essentially faced with reaching agreement on a single issue.[13] It is assumed that individual i's estimate of parameter i^* prior to the deliberation process (i.e. at time $t = 0$) is given by the noisy signal

$$\mathsf{i}_i^0 = \mathsf{i}^* + \epsilon_i, \qquad \epsilon_i = iid(0, \sigma^2) \tag{8}$$

Further, member j is assigned initial *precision* π_{ij}^0 by individual i, namely

$$\pi_{ij}^0 = var_i\,[\epsilon_j]^{-1} \tag{9}$$

According to this structure, $\lim_{\epsilon_j \to \infty} = 0$. In other words, the greater the variance associated with j's information, the smaller the associated precision. It is noted that in their original paper, DVZ assume introduce a binary variable $q_{i,j}$ where

$$q_{i,j} = \begin{cases} 1, & \text{if } i \text{ listens to } j; \\ 0, & \text{if } i \text{ does not listen to } j. \end{cases} \tag{10}$$

I assume that because MPCs are composed of members who meet *face to face* on a regular basis, it is not feasible to suppose that members *cannot* listen to each other. Therefore I impose the assumption that $q_{i,j} = 1$ for all members. Unlike DG, the weight which members of a committee place on the opinions of others is determined by members' *precisions*, with the precision assigned to another member's information being a members *subjective assessment*. This may emanate from a member's disclosure of private information, $\Lambda_{t,j}$. Only certain members of the committee have information which is worth listening to, and the more useful the information the smaller the variance. Information deemed useless is assigned an infinitely large variance.

More formally, for an m member monetary policy committee, let the weight given by a member c to the information of a member k be given by $\pi_{c,k}^0 / \sum_{j=1}^{m} \pi_{c,j}^0$. The sum of these weights is necessarily one, and in line with in DG, these weightings are then assigned to an $m \times m$ matrix T, where the weighting $\pi_{c,k}^0 / \sum_{j=1}^{m} \pi_{c,j}^0$ corresponds to the element in the c^{th} row and k^{th} column. Matrix T thus has the characteristics of a transition matrix, the elements in each row being non-negative and summing to unity. For completeness, T is more explicitly expressed as

$$\mathsf{T} = \begin{bmatrix} \frac{\pi_{1,1}^0}{\sum_{j=1}^{m} \pi_{1,j}^0} & \cdot\ \cdot\ \cdot & \frac{\pi_{1,m}^0}{\sum_{j=1}^{m} \pi_{1,j}^0} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \frac{\pi_{m,1}^0}{\sum_{j=1}^{m} \pi_{m,j}^0} & \cdot\ \cdot\ \cdot & \frac{\pi_{m,m}^0}{\sum_{j=1}^{m} \pi_{m,j}^0} \end{bmatrix} \tag{11}$$

[13] This is unlike the DVZ set-up, where there were many issues.

Updating occurs is similar fashion to DG, only in this instance the belief vector containing members' interest-rate preferences prior to the deliberation process is populated by noisy estimates of i^*. Again, this vector contains the interest rates members would choose to set were they given *individual* responsibility for monetary policy. Information pertaining to individuals' initial signals is then communicated to each other through a *social network*, and members update their views in contiguous deliberative rounds. Updating in the first round takes place according to by post-multiplying the listening matrix by member's initial interest-rate beliefs. First round revisions are thus explicitly defined as

$$\mathbf{I}^1 = \begin{bmatrix} \frac{\pi^0_{1,1}}{\sum_{j=1}^{m}\pi^0_{1,j}} & \cdots & \frac{\pi^0_{1,m}}{\sum_{j=1}^{m}\pi^0_{1,j}} \\ \vdots & \ddots & \vdots \\ \frac{\pi^0_{m,1}}{\sum_{j=1}^{m}\pi^0_{m,j}} & \cdots & \frac{\pi^0_{m,m}}{\sum_{j=1}^{m}\pi^0_{m,j}} \end{bmatrix} \times \begin{bmatrix} \mathsf{i}^0_1 \\ \vdots \\ \mathsf{i}^0_m \end{bmatrix} \tag{12}$$

$$= \begin{bmatrix} \sum_j \frac{\pi^0_{1,j}}{\sum_{j=1}^{m}\pi^0_{1,j}}\mathsf{i}^0_j \\ \vdots \\ \sum_j \frac{\pi^0_{m,j}}{\sum_{j=1}^{m}\pi^0_{m,j}}\mathsf{i}^0_j \end{bmatrix} \tag{13}$$

As is the case with the DG model, if members' revised rates have not converged to the same interest-rate in the first period, the revision process continues until it does with agents treating the information in each round as new and independent. Up to the n^{th} period revised opinions are calculated as

$$\begin{aligned} \mathbf{I}^{[1]} &= \mathsf{T}\mathbf{I}^{[0]} = \mathsf{T}^1\mathbf{I}^{[0]} \\ \mathbf{I}^{[2]} &= \mathsf{T}\mathbf{I}^{[1]} = \mathsf{T}^2\mathbf{I}^{[0]} \\ &\vdots \\ \mathbf{I}^{[n]} &= \mathsf{T}\mathbf{I}^{[n-1]} = \mathsf{T}^n\mathbf{I}^{[0]} \end{aligned} \tag{14}$$

where T^n is the matrix T raised to the n^{th} power.

3.3 Graph Theoretic Representation of Direct and Indirect Influence

The nature of *direct* and *indirect influence* in the model can be depicted using *graph-theoretic representation*. In terms of graph theory, a graph is a structure comprised of *nodes*, which represent members of a group, and lines connecting the nodes together, known as *edges*.[14] Edges are *directed*

[14] Sometimes *nodes* and *edges* are referred to as *vertives* and *arcs* respectively.

- giving them the appearance of arrows - which means that it is possible to capture the direction of influence running from member to member. For example, a directed edge running directly from member j to member k indicates that j is directly influenced by k. Put another way, element $\mathsf{p}_{j,k} \in (0, 1]$. However, if k is *also* directly influenced by j, both members will be connected by an arrow, $\leftrightarrow$, which runs in both directions.

Figure 4.0 gives simple examples of possible influence structures. Diagram (i) can be construed as a special case of a *one* member committee where that member listens only to himself. The unidirectional arrow running from the node to itself is thus used to show that j directly influences himself. A member *directly influencing* himself is represented using a unidirectional arrow running from a node to itself. This is illustrated in (i) for the case of a member j. For this member $p_{jj} = 1$ - (ii) depicts a two member group comprised of j and k, who do not communicate with each other - both members effectively ignore each other, listening to their own opinion only. Therefore, $p_{jk}, p_{kj} = 0$ and $p_{jj}, p_{kk} = 1$. Parts (iii) and (iv) depict two member groups where individuals are influenced *directly* by each other. In (iii) member j is directly influenced by k, although k ignores j, choosing to listen to himself only. Therefore $p_{jj}, p_{kj} = 0$ and $p_{jk}, p_{kk} = 1$. In (iv) both group members listen to themselves and are *directly influenced* by each other. Therefore it must follow that $p_{jj}, p_{kj}, p_{jk}, p_{kk} \in (0, 1)$. The final example, ($v$) depicts the nature of *indirect* influence amongst a three member group, and shows that although j is *indirectly influenced* by k, the opposite cannot be said to be true. The example demonstrates that with only a three member network, the nature of relationships between its members can be complex.

The notions of direct and indirect influence can also be mapped to standard concepts in the theory of Markov chains. A *transient state* has a steady-state probability of zero. In FIGURE 4.0 **(iii)** j is equivalent to a transient state: once j reaches k it is is impossible to get back. A *transient set* contains a group of states all of which have steady-state values of zero. A *recurrent set* contains a set of states such that once the system enters it, it always makes transitions within the set and never leaves it. In FIGURE 4.0 **(v)** l and k form a recurrent set. Once j enters it, it is impossible to return. An *absorbing state* is a special case of a recurrent set which contains only one state. This is the case in FIGURE 4.0 **(iii)** - k is an absorbing state. If the entire system is a is a recurrent set, then it is called *ergodic*. FIGURE 4.0 **(iv)** is an example of an ergodic system. If a system is not *ergodic*, then there may be more than one recurrent set in the system. FIGURE 4.0 **(v)** is therefore *not* ergodic - it contains one recurrent set (k and l) and a transient state (j), and is characteristic of an *absorbing chain.*

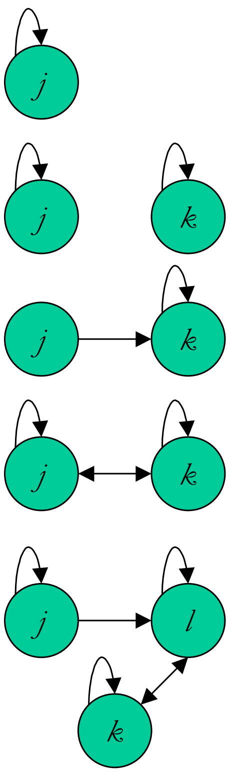
4 Diverse Interest-Rate Preferences

Each MPC member is assumed to have a preferred interest rate which is *non-negative* and *continuous*, and predominantly based on all currently available economic information.[15] More formally, write that

$$i^*_{j,t} \in \{0 \leq i < \infty\} = f(\Omega_t, \Lambda_{t,j}, M_j) \tag{15}$$

Equation (15) represents member $j's$ choice of interest-rate i_j as a function of *group information* known to all committee members in the current period, Ω_t, plus information specific to that in-

[15] In practice however, the values from which a member will choose from are likely to be constrained to a small finite set of values.



(i) *j directly influences* himself.

(ii) *j* and *k* do **not** *influence* each other. *j directly influences* himself, as does *k*.

(iii) *j* is *directly influenced* by *k*, but does not *influence* himself. *k* is not *influenced* by *j*. *k directly influences* himself only.

(iv) *j* and *k* both *directly influence* each other. *j* is *directly influenced* by himself, as is *k*.

(v) *j* is *indirectly influenced* by *k* via *l*. However, *k* is not *indirectly influenced* by *j* because *l* does not *influence j*. *l* and *k directly influence* each other. Neither *j* nor *k directly influence* each other.

Figure 4.0.
Graph theoretic representations of direct and indirect influence

dividual member, $\Lambda_{j,t}$. Refer to member specific information as member j'^s *private information.* This general framework reflects real-world features of monetary policy decision making. For example, in the case of the Bank of England, Ω_t may be construed as embodying information contained in internal Bank forecasts and the numerous economic analyses presented to MPC members at pre-MPC meetings by bank staff. This information is common knowledge to all MPC members.[16] The existence of private information, $\Lambda_{j,t}$, emanates from the fact that different members will invariably be both exposed to and receptive to different sources of information during their time on the MPC.[17] In addition to the role of information, it is supposed that MPC members may have different views of how the economy works, and to put it crudely, different models of the economy in their heads.[18] Thus, even when members are presented with the same economic facts, they may still prescribe conflicting courses of action for the economy - different members may treat the same information *differently.* In light of this, M_j denotes a member j's model of the economy.[19] The framework embodied in (15) is also assumed to extend to the United States FOMC and the ECB Governing Council. However, having arrived at a view as to the most appropriate interest-rate, members must now arrive at a collective decision. The mechanism which captures this process is described in the following section.

4.1 Weighting the Opinions of MPC Members

Given $\Omega_t, \Lambda_{t,j}$ and M_j there is no guarantee that members $j = \{1, 2, .., J\}$ will prescribe the same interest-rate for the economy. Indeed, prior to the deliberation process, members may initially hold different views. In this framework, (15) accounts for such diverse interest-rate preferences. Where differences in opinion arise, some mechanism exists which brings about a group consensus or sufficient agreement leading to a majority of votes being cast in favour of one particular course of action. Having formed an opinion on the interest-rate, members are assumed to reach an agreement on the interest-rate using the iterative procedure outlined in (7). A key issue, however, is MPC members' criteria for allocating weights. Some weighting allocations will lead to *no* consensus being reached. Yet for the FOMC, ECB Governing Council and Bank of England Monetary Policy Committee, failure to reach a decision is emphatically *not* an option - a course of action must be

[16] Much of this information is not in the public domain. For example, the monthy report on regional developments presented at pre-MPC meetings by the Bank of England's regional agencies is unavailable to the public at large. In this sense, the information forming MPC members' common information set is still exclusive to the monetary authorities.

[17] In the case of European monetary union, a member j of the ECB's governing council may not divulge private information which, if it becomes common knowledge, may lead to other committee members taking an interest rate decision detrimental to member j^s economy.

[18] Sushil Wadhwani (2002) is an example of an MPC member to openly express doubts about the efficacy of the suite of Bank of England models used in the forecast of inflation and GDP. These forecasts, which form the basis of the Bank's *Inflation Report*, are purportedly integral to decisions on the interest rate, and are supposed to represent the 'collective judgement' of the MPC.

[19] This has a clear analogy with jury deliberations. Even when presented with exactly the same evidence in a trial, jurors arrive at different conclusions as to the innocence or guilt of a defendant. Although this may be due to some jurors having a lower threshold of doubt than others, it is not implausible to assume that members may use different lines of reasoning than their fellow jurors or attach different levels of importance to the same piece of evidence. In much the same way, committee members may not weight all economic information in the same way when arriving at a decision about monetary policy. Evidence that this occurs is given by Goodhart (1999), stating that

> "What...is the current sign of the output gap? As evidenced by our differing votes, we in the MPC can and do see the same underlying data having different implications for that gap." (pp.247-248)

agreed upon.[20] Such results are in a sense not plausible.

4.2 Allocating Weights to Members' Opinions

A clear obstacle in applying this model lies in the allocation of weights of others' opinions. Weights are not determined arbitrarily, but determined by (*i*) career concerns, a view which is not entirely new to the literature and (*ii*) the precision of members' information, as exemplified by the general approach of DVZ. In relation to (*i*), consider Havrilesky and Schweitzer's (1990) proposal that dissent voting is a source of disutility for individuals serving on the US FOMC. When differences in opinion arise, the presence of career concerns can lead a member to 'revise' their view and subsequently vote with individuals who are perceived to have a bearing on his or her career path. Lawrence Roos, president of the St. Louis Federal Reserve Bank pays testament to this view, forging a clear link between dissent voting behaviour and career concerns:

> "If one is a young, career oriented president who's got a family to feed he tends to be more moderate in his opposition to governors."[21]

The message here seems to be that even when views on the interest-rate are divergent, social forces come into play thus bringing about agreement amongst committee members. Accordingly, members are more likely to pay attention to members of a similar type as they may have to work with them in the future - perhaps even long after their time on serving on the committee has ended. Disagreement *now* may hamper future prospects of promotion, and although members care about the *objectives* of the committee, they also care about future *career advancement*. In this sense, the way a member 'performs' on an MPC is not just measured as their ability to hit an inflation target, so to speak, but judged according to a their propensity to agree with members of his or her type.

This may be particularly true for *insiders* serving on the Bank of England MPC. As stated in the section in *individualistic* MPCs, the voting record for the first five years of the MPC, demonstrates that approximately $16\frac{1}{2}\%$ of members' votes were dissents. Essentially, MPC members are twice as likely to dissent than FOMC members. Yet this statistic can be broken down further. Bank *insiders* were on the winning side of a monetary policy decision 93.6% of the time, compared with outsiders for whom the figure was 73.9%. In other words, insiders are far less likely to dissent than outsiders, although when insiders dissent, they do so on the side of monetary tightness, whereas outsiders do so on the side of monetary ease. Yet an equally compelling finding is that *insiders* were in agreement with each other more so than *outsiders*. To quote Edmonds (1999) insiders behaved as if they were a "cohesive homogeneous group."[22]

To substantiate this claim, I develop the following measure, Θ_j, which corresponds degree of agreement between *insiders* and *outsiders* respectively, at the average MPC meeting. Let this be

[20] Failure to reach a decision would severely damage the reputation on the monetary authorities.

[21] *cf.* Havrilesky and Schweitzer (1990), p.3. Also considered is the opinion of Henry Wallich, a member of the Board of Governors:

> "It is not a pleasant thing to have to keep dissenting...One dissents less often than you would think. After all you are a member of a group and you want to get along with the other members."

[22] Edmonds (1999), p.3.

expressed as

$$\Theta_j = \frac{1}{T}\sum_{t=1}^{T}\frac{\sigma_{j,t}}{\bar{i}_{j,t}}, \qquad j = I, O \tag{16}$$

where I and O denote insiders and outsiders respectively, $\sigma_{j,t}$ denotes the standard deviation of the desired interest-rate for members of group j in meeting t, and $\bar{i}_{j,t}$ the corresponding mean interest-rate for each group in meeting t. The term $\frac{\sigma_{j,t}}{\bar{i}_{j,t}}$ is no more than the coefficient of variation in meeting t. Lastly, let T stand for the total number of meetings included in the sample. Essentially, I calculate the *coefficient of variation* of interest rate preferences for each group at each meeting, and then take the group average across *all* meetings in the sample, to proxy for the measure of agreement within each group *at the average meeting.*. Accordingly, the smaller the value of Θ_j, the lower the level of variability in interest-rate preferences expressed by members at each meeting. A value of $\Theta_I = 0$ would suggest that insiders were on average in perfect agreement with each other in every MPC meeting. The coefficient of variation associated with outsider preferences over the interest-rate - $\Theta_O = 1.508$ - is somewhat higher than those of insiders, for whom $\Theta_I = 0.563$. The policy preferences of outsiders at the average meeting are more diverse that those of *insiders*, who on average, have more of a tendency to agree with each other at each MPC meeting.[23] Indeed, insiders were in total agreement with each other in 56 out of 74 meetings (i.e. when $\frac{\sigma_{I,t}}{\bar{i}_{I,t}} = 0$). For outsiders, the number was in only 31 meetings (i.e. when $\frac{\sigma_{O,t}}{\bar{i}_{O,t}} = 0$). These results are plotted in FIGURE 4.3.

[23] Further, the finding that *insiders* are more likely to agree with each other more than *outsiders* has important ramifications for the way the MPC is perceived. It is commonplace in the financial press and media to portray insiders and outsiders as two distinct groups, each with well defined objectives and goals. These result serve to partially dispel this myth.

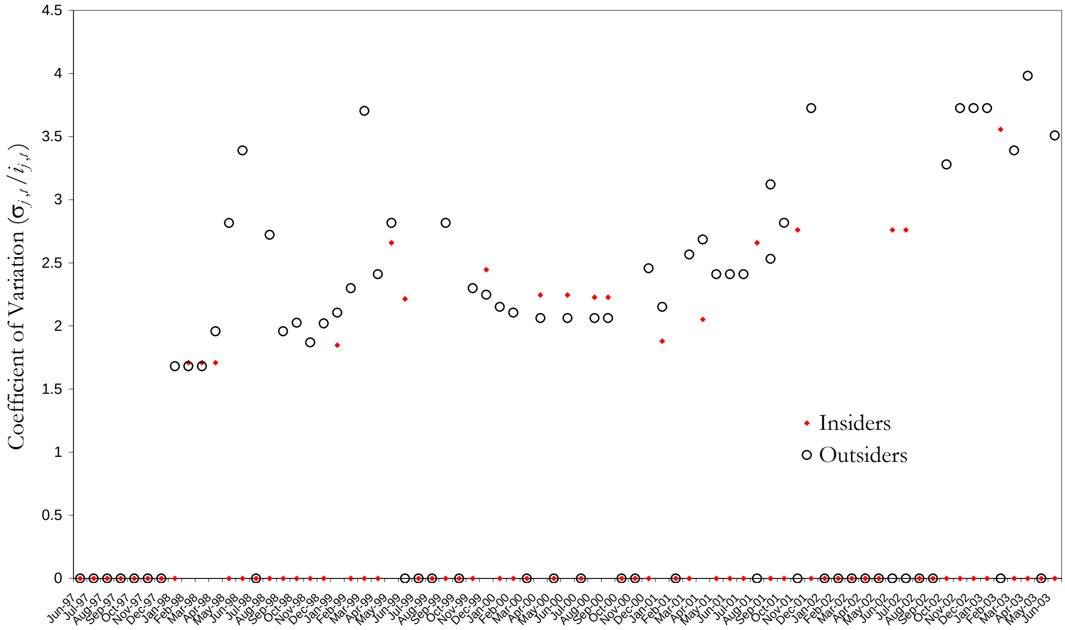


FIGURE 4.3:
COEFFICIENT OF VARIATION FOR INTEREST RATE PREFERENCES OF INSIDERS AND OUTSIDERS AT EACH MONTHLY MPC MEETING, JUNE 1997 - JUNE 2003

Whereas insiders may be seen to have well defined goals and form a homogeneous cohesive group, outsiders form more of a disparate grouping. I suggest that such observed voting behaviour arises due to *insiders* being incentivised to pay more attention to each other than outsiders. In the context of the model, this leads to insiders only weighting the policy preferences of fellow insiders in the course of MPC deliberations, which can be explained by the presence of career concerns.[24,25] As insiders have a 5-4 in-built majority over outsiders one might reasonably posit that if *insiders* voted together as a group - essentially weighting only the opinions of other insiders - they would dominate decisions on the MPC, regardless of any opposition from their externally appointed counterparts. *This is what is seen in practice.*

[24] This may be because, as Buiter (1999) points out, the appointment of *insiders* to the MPC emanates from their positions in the hierarchy of the Bank. Given the fact that insiders enjoy a numerical majority on the MPC, it is reasonable to suppose that were an 'institutional consensus' to develop - which is likely in an organisation like the Bank, which has a strong internal culture and sense of 'corporate identity' - *insiders* would dominate MPC decisions by dint of their inbuilt majority. Put another way,

> '...every organisation develops, in short order, an in-house view, an orthodoxy, a conventional wisdom, which it becomes increasingly difficult to challenge' (Buiter (1999), p.13.)

For this reason Buiter suggests that it would be apposite for the committee to have a majority of *outsiders*, or rather, individuals who do not hail from a single organisation.

[25] In the event of a split decision, the Governor of the Bank in his role as Chairman of the MPC has a casting vote.

5 Unanimous versus Majority Consensus

I now define two types of consensus which can be reached by a committee: *unanimous consensus* and *majority consensus*. A *unanimous consensus* occurs where members' opinions all converge on the same interest-rate. Contrast this with a *majority consensus*, where at least half of all elements in the *belief* vector converge to the same point estimate in the limit. The former category is most immediately applicable to *autocratically-collegial* and *genuinely-collegial* monetary policy committees - in both of these committees, the ability to reach a deliberative outcome characterised by all members being in agreement with each other. Unanimity is highly valued, and thus consciously sought. As a real-world example of *unanimous consensus*, recall the remarks of Wim Duisenberg at an ECB press conference following the monetary policy decision made by the Governing Council on February 3^{rd} 2000:

> "First, there was no formal vote. Again....it was a *consensus* decision." (*emphasis added*)

The statement implies that all members of the ECB Governing Council were agreed on the same interest-rate. Therefore its members can be said to have reached a *unanimous consensus*. A *unanimous consensus* might also apply to an *individualistic* committees where all members vote in favour of the Chairman's policy proposal, such as the August 2002 meeting of the Bank of England MPC - all members voted in favour of the proposition that the Bank's repo-rate should be maintained at 4%. However, a *majority consensus* is more likely to be characteristic of the decisions reached by *individualistic committees*, where agreement amongst *all* members is not sought or expected. As previously attested, the nine member Monetary Policy Committee of the Bank of England is such a body, and this is borne out in the previous section.

5.1 Convergence to a Unanimous Consensus

A *unanimous consensus* is reached if *all* elements in the belief vector converge to the same value in the limit as $n \to \infty$. More formally, consensus is achieved by all MPC members if

$$\lim_{n\to\infty} \mathsf{i}_{j,n} = \mathsf{i}^*_{j,n} \qquad \forall\ j = 1, 2, ..., m. \tag{17}$$

This only occurs where there is a $(1 \times m)$ row vector $\boldsymbol{\pi} = [\pi_1, ..., \pi_m]$ such that for $j = (1, 2, ..., m)$ and $l = (1, 2, ..., m)$,

$$\lim_{n\to\infty} \mathsf{p}^{(n)}_{j,l} = \boldsymbol{\pi}_l \tag{18}$$

where $\mathsf{p}^{(n)}_{j,l}$ is an element belonging to the transition matrix P^n from row j and column l. In other words, for an m member committee, *unanimous consensus* is achieved when the elements of $\mathsf{P}^{n\to\infty}$ converge on a distribution characterised by m identical rows; in any given column, the elements will be identical. Further, in keeping with the properties of a transition matrix, the elements will be non-negative and sum to unity, specifically

$$\sum_{j=1}^{m} \boldsymbol{\pi}_j = 1 \tag{19}$$

5.2 Convergence to a Majority Consensus

A *majority consensus* is reached if at least half of all elements in the belief vector converge to the same value in the limit as $n \to \infty$. More formally, consensus is achieved amongst a majority of MPC members if

$$\lim_{n\to\infty} \mathsf{i}_{j,n} = \mathsf{i}^*_{j,n} \qquad \forall\ j \neq k,\ k = \{1, 2, ..., \frac{m-1}{2}\} \tag{20}$$

where

$$\lim_{n\to\infty} \mathsf{i}_{k,n} \neq \mathsf{i}^*_{j,n} \qquad \forall\ k \tag{21}$$

This will only occur when there is a vector $\boldsymbol{\pi}_j = [\pi_1, ..., \pi_m]$ such that for $j, l = (1, 2, ..., m)$,

$$\lim_{n\to\infty} \mathsf{p}^{(n)}_{j,l} = \boldsymbol{\pi}_l, \qquad j, l \neq k \tag{22}$$

where $\mathsf{p}^{(n)}_{j,l}$ is an element belonging to the transition matrix P^n from row j and column l. In other words, for an m member committee, *majority consensus* is achieved when the elements of $\mathsf{P}^{n\to\infty}$ converge on a distribution characterised by $j \geq \frac{m+1}{2}$ identical rows;[26] unlike *unanimous consensus*, all that is required here is for a majority of columns, the elements will be identical. As was specified for unanimous consensus, under *majority consensus* the elements in each row will be non-negative and sum to unity.

6 Conditions for Reaching a Consensus

In specifying the conditions for reaching a consensus I develop theorems whose corresponding proofs have a basis in the theory of Markov chains. Such proofs can be found in standard reference texts on stochastic processes, probability theory, matrix theory and finite Markov chains.[27] It is assumed throughout that *no two members share the same initial policy preferences.* This implies that each element of the belief vector is unique.[28] Conditions under which *unanimous consensus* is achieved are now stated.

6.1 Unanimous Consensus

Proposition 1: A unanimous consensus will be reached by the committee if the influence matrix P is irreducible and aperiodic.

[26] To keep the analysis simple, odd m is assumed.

[27] See for example Doob (1953), Feller (1968), Karlin (1969), Parzen (1962), Berman and Plemmons (1979), Kemeny and Snell (1960) and Theil (1972).

[28] This is an important assumption, and has ramifications for the conditions under which consensus will be reached. Here, the work of Berger (1981) is of particular importance. In a follow up paper to DG, Berger corrects DG's assertion regarding the necessary conditions for reaching a consensus. Simply put, whereas DG asserts that whether a consensus can be reached depends only on the influence matrix, P, Berger shows how it is also dependent on the initial (time $t = 0$) composition of the *belief vector*, $\mathsf{I}^{[0]}$. For example, assume that prior to the deliberation process all elements of the belief vector are equal, that is $\mathsf{i}_{1,0} = \mathsf{i}_{2,0}...\mathsf{i}_{m-1,0} = \mathsf{i}_{m,0}$, $n = 1, 2, ...\infty$. The make-up of P is hence irrelevant and a *unanimous consensus* is reached as $\mathsf{I}^{[n]} = \mathsf{P}^{[n]}\mathsf{I} = \mathsf{I}^{[0]}$. Given that the concern of this paper is to determine how members of an MPC rach agreement under initially diverse interest rate preferences, I do not consider this prospect.

The following definition is introduced:

Definition 1: A transition matrix is irreducible if and only if for every (j,k) there exists a natural number q such that

$$p^{q}_{j,k} \in (0,1) \tag{23}$$

In other words, if *all* elements in the influence matrix are positive for some power q, it has a unique long-run stationary distribution. In other words, although the transition matrix $\mathsf{P}^{[0]}$ may contain some zeros, members are sufficiently connected such that *every member listens to every other member either directly or indirectly.* When the matrix is raised to some power q, all of the elements in the listening matrix become strictly positive but less than unity. It is crucial to note here that no $p^{0}_{j,k} = 1$, which would imply *periodicity* or an *absorbing class.* By (23) all recurrent states of the Markov chain *communicate* with each other and are *aperiodic*. A transition matrix with such properties is sometimes called *regular.*

Proof of Proposition 1: *See Appendix.*

The concept of *irreducibility* has an equivalent *graph theoretic* interpretation. If the *digraph* corresponding to the matrix P is *strongly connected*, then it is *irreducible. Strongly connected* asserts that for any ordered pair (P_i,P_j) of vertices there exists a sequence of paths leading from P_i to P_j. Monetary policy committees whose members are strongly connected will necessarily reach a consensus. Simulated examples of strongly connected MPCs are provided in proceeding sections. I now introduce the following further proposition relating to *unanimous consensus.*

Proposition 2: A unanimous consensus will be reached by the committee if any member j is influenced only by himself, and influences all other members, either directly or indirectly. Members' beliefs will necessarily converge to those of member j.

Proof of Proposition 2: *See Appendix.*

Such a member j is akin to monetary policy dictator of the variety assumed to yield influence in *autocratically-collegiate* MPCs.

6.2 Majority Consensus

In this section I restrict myself to the case where there are two distinct groups of members on a monetary policy committee. This set-up is analogous to the institutional arrangement at the Bank of England, and in line with the terminology popularly used to describe members of its monetary policy committee, I refer to them as insiders and outsiders. This leads to Proposition 3.

Proposition 3: If there exists two distinct groups of members within a monetary policy committee who do not communicate with each other, and each group forms an aperiodic recurrent class, for an m member committee, a majority consensus will be reached by the group with the largest number of members.

Proof of Proposition 3: *See Appendix.*

The intuition underlying Proposition 3 is easy to understand. It says that if *insiders* and *outsiders* listen only to members of their own type (thereby assigning zero weight to the views of those members not of their type) insiders will be neither *directly* nor *indirectly* influenced by outsiders, and vice versa. By assuming that insiders and outsiders each form an *aperiodic recurrent class*, it follows that members in each group will reach a consensus amongst themselves. In the presence of initially diverse interest rate preferences, it is still possible for members to reach a unanimous consensus if the consensus rate of interest for both converges on the same rate. As a special case, this is demonstrated in the appendix, and follows the proof of Proposition 3. Insiders are assumed to listen only to insiders, with the equivalent being true for outsiders.

6.3 Failure to Reach a Consensus

I now introduce the following proposition, which underscores the importance of listening to other MPC members when views are initially diverse.

Proposition 4: If no MPC member is directly or indirectly influenced by any other member, a consensus will never be reached.

Proof of Proposition 4: *See Appendix.*

This result follows because in each stage of the deliberation process each member only updates his beliefs on the basis of listening to himself. Consequently, a given member's beliefs remain unchanged from period to period. This result suggests that MPCs should be populated by people who are willing to listen to the views of others. A committee comprised of 'egoists' will never reach agreement because in not weighting the opinions of others, members are unable to budge from their initial positions on the interest-rate. If the assumption of initial belief diversity is relaxed (i.e. some members share the same interest rate preferences prior to the deliberation process), a *majority consensus* will be reached if at least half of, but not all members share the same initial belief. When all members share the same initial belief a *unanimous consensus* is achieved. In either case there is no need to deliberate.

6.4 Periodic Behaviour

Not all *irreducible* Markov chains thus have the property of *ergodicity*, and It is possible for a Markov chain to be both irreducible *and* periodic. Assuming initially diverse interest rate preferences, periodicity implies that $\lim_{n\to\infty}\mathsf{P}$ is not characterised by a *unique stationary distribution.* A periodic Markov chain with n eigenvalues of modulus 1 has period n. In terms of the listening matrix P, this implies that members may be only be *directly* or *indirectly influenced* by other members at regular intervals (i.e. every n^{th} interval). From an intuitive perspective, such behaviour is unappealing. Periodicity implies that members constantly switch to listening to different members at different stages of the deliberation process. I nevertheless introduce the following proposition:

Proposition 5: Under initially diverse interest rate preferences and assuming periodicity, neither a unanimous nor majority consensus will be achievable when states are periodic.

Proof of Proposition 5: *See Appendix.*

It is noted that the results for Proposition 5 do not extend to the case where the influence matrix P is characterised by m sets of *cyclic recurrent classes.* Under such conditions, it is possible that (i) there exist some linear combination of opinion weights and initial beliefs ensuring that $\mathsf{I}^{[1]} = \mathsf{I}^{[2]} = ... = \mathsf{I}^{[n]}$ (*unanimous consensus*) or (ii) a majority of members beliefs converge on the same interest rate for $\mathsf{I}^{[n]}$, $n = 1, 2, ...\infty$ with the *composition* of the majority switching regularly in sync with the period of the cycle (*majority consensus*). Illustrative examples of such behaviour proceeds the proof of Proposition 5 in the appendix. I now turn to the simulations.

6.5 Speed of Convergence to a Consensus

I have not yet touched on the *speed* at which beliefs will converge. In any transition matrix which is irreducible and aperiodic (therefore entailing that a *unanimous consensus* is reachable) the absolute value of the largest eigenvalue will equal unity, with the corresponding moduli of *all* other eigenvalues being smaller than one. Indeed, eigenvalue analysis may be used to check whether convergence occurs, and a sufficient condition for reaching a unanimous consensus characterised by a *unique stationary distribution* is given by the case where all other $m-1$ eigenvalues have moduli strictly less than one. The *rate* with which convergence is achieved is related to the *second largest eigenvalue in absolute value.* Define this as

$$\delta(P) = \max\{|\lambda| : \lambda \in \sigma(P)\}, \lambda \neq 1\} \tag{24}$$

where $\sigma(P)$ is the set of all eigenvalues in P. $\delta(P)$ declines geometrically with each iteration. For some transition matrices, it is possible that the roots may be *complex.* Calculating the absolute value of a complex number is given by the square root of its modulus. Therefore, for any complex eigenvalue $\lambda_j = a \pm bi$, its absolute value can be defined as

$$|\lambda_j| = (|a \pm bi|)^{\frac{1}{2}} = (a^2 + b^2)^{\frac{1}{2}} \tag{25}$$

Should the second largest eigenvalue λ_2 be complex, the convergence towards the stationary distribution is of the *damped oscillary* type. Further, if two influence matrices $\widetilde{P}$ and $\widetilde{\widetilde{P}}$ are compared such that the $\delta(\widetilde{P}) < \delta(\widetilde{\widetilde{P}})$, then consensus will be reached more quickly under $\widetilde{P}$ than for $\widetilde{\widetilde{P}}$. It is noted that the largest eigenvalue of *any* transition matrix always is always a unit root. The presence of more than one eigenvalue of modulus 1 does not necessarily imply that a *unanimous consensus* has not been reached, as this will also depend on the initial elements of the of *belief vector.* In the case of a majority consensus, there will be at least two eigenvalues with moduli equal to 1.

It is possible to use $\delta(P)$ to determine the number of iterations until consensus is reached. A 'rough and ready' estimate is given by the formula

$$\xi = \frac{\log 0.008}{\log(\delta(P))} \tag{26}$$

where ξ is the number of iterations (i.e. deliberative rounds) and logarithms are to base 10.

7 Simulation Results

PANELS 4.1-4.4 show various influence structures for MPCs. PANEL 4.1 **(a)** shows the influence network for a five member monetary policy committee where each member places positive weight on the opinions of all members, including himself. In line with the institutional arrangements at the Bank of England, two types of member are specified - *insiders* and *outsiders*.[29] Further, the committee comprises three of the former type and two of the latter type.[30] Specifically, MPC members' ideal interest rates are captured by the belief vector $\mathsf{I}^{*,0}$,

$$\mathsf{I}^{*,0} = \begin{bmatrix} 4.5 \\ 4.4 \\ 4.3 \\ 4.2 \\ 4.1 \end{bmatrix} \begin{matrix} \text{MEMBER} \\ \textsf{Governor} \\ \textsf{Insider 1} \\ \textsf{Insider 2} \\ \textsf{Outsider 1} \\ \textsf{Outsider 2} \end{matrix} \tag{27}$$

and *opinion weights* in the transition matrix P is are allocated such that

$$\mathsf{P} = \begin{bmatrix} 0.8 & 0.05 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.8 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.8 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.8 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.8 \end{bmatrix} \begin{matrix} \text{MEMBER} \\ \textsf{Governor} \\ \textsf{Insider 1} \\ \textsf{Insider 2} \\ \textsf{Outsider 1} \\ \textsf{Outsider 2} \end{matrix} \tag{28}$$

In P, rows 1-3 contain the opinion weights pertaining to insiders, and the remaining two rows contain outsiders' allocation of weights. Further, assume that the insider in the first row is the Governor, who also assumes the role of committee Chair. The general configuration of (28) applies to all simulations. In this example, a *unanimous consensus* is achieved as *all* members have a direct influence on each other. The MPC is strongly connected, with the influence matrix thus being *irreducible* and *aperiodic*. Corresponding numerical parameters for the opinion matrix are also shown. The speed of convergence, $\delta(P) = 0.75$. Using (26) it is determined that the group reaches a consensus after approximately $\xi = \frac{\log 0.008}{\log 0.75} = 16.78$ deliberative rounds. Such an influence structure structure may correspond to that at the European Central bank, and more generally, *genuinely collegiate* monetary policy committees.

In PANEL 4.1 **(b)**, each member only listen to himself. By application of Proposition 4, because no MPC member is directly or indirectly influenced by any other member a consensus will never be reached. It is notable that the group *would* reach a majority or unanimous consensus if a majority of or all committee members were endowed with same initial interest rate beliefs (i.e. if the assumption of initially diverse beliefs is relaxed). No agreement is otherwise reached, as is borne out in the example.

In PANEL 4.2 **(c)**, each member only weights the opinion of a single member other than himself. No two members weight the opinion of the same member, and P is *irreducible* and *periodic* with

[29] Alternatively, one could consider the distinction between Board members and Bank Presidents on the Federal Open Market Committee or National Central Bank Governors and Board members sitting on the European Central Bank's Governing Council.

[30] As is the case with the MPC, it is supposed that *insiders* comprise the majority of members.

period 5. From Proposition 5, neither unanimous nor majority consensus are reachable, and associated transition matrix has no stationary distribution. This configuration has little intuitive appeal, as members' beliefs follow a cyclical pattern switching abruptly from period to period. However, it is notable that unanimous consensus is reachable *if and only if* the elements of the belief vector are initially equal. A *majority consensus* is possible if a majority of elements in belief vector are equal, although the composition of the majority will switches from period to period following a cycle. PANEL 4.2 **(d)** depicts a monetary policy committee of insiders and outsiders who do not communicate with each other. The example shows that even when both groups ignore each other consensus is still reachable, even under initially diverse interest rate preferences. It constitutes the special case of Proposition 3, as elaborated on in the proof given in the appendix.

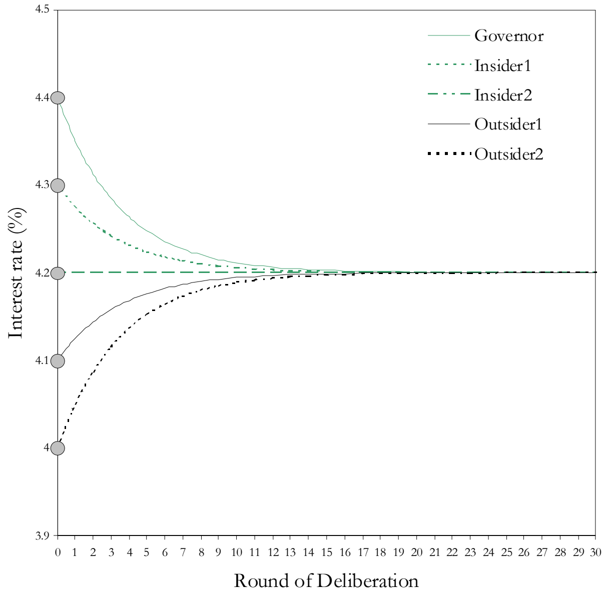
In PANEL 4.3 **(e)**, four members weight only the opinion of a single member, in this case the Governor. As the Governor only weights his own opinion, the interest rate converges to his preferred rate. A unanimous consensus is thus reached. This is an example of an absorbing chain, the proof of which is given in Proposition 2. The example corresponds to a structure like the FOMC, which is an example of an autocratically collegiate committee. Here, all members weight the opinion of the Governor, but he does not reciprocate, opting to weight his own opinion only. In PANEL 4.3 **(f)** members are partitioned into two groups - insiders and outsiders. However, if two members – one from each group - both communicate with each other, a unanimous consensus is reached. This is because from Proposition 1, the influence matrix is both *aperiodic* and *irreducible*.

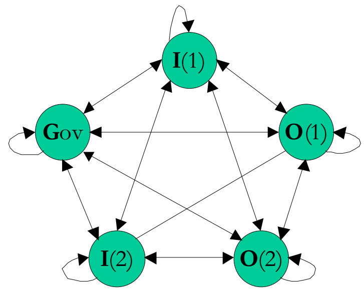
The final two simulations shown in PANEL 4.4 depict possible states of affairs for an *individualistic* MPC. Both examples are geared specifically towards the institutional nuances of the Bank of England Monetary Policy Committee, and reproduce the stylised facts of voting behaviour associated with its members. In line with Proposition 3, insiders and outsiders are seen to reach a consensus amongst themselves. However, it is assumed that outsiders initially prefer lower interest rates than outsiders. When insiders vote together as a group, they are seen to dominate decisions on the MPC, regardless of any opposition from their externally appointed counterparts. Insiders reach a *majority consensus*, because they restrict themselves to only weighting the opinions of members of their own type. Further, the policy outcome is typified by an interest-rate which is higher than would be chosen by outsiders.[31]

The two examples also illustrate the importance of *listening to others* in the course of MPC deliberations. **(g)** can be viewed as comprising a committee of 'pragmatists', whereas **(h)** comprises a committee of 'egoists'. Begin by assuming that the belief vector containing member's ideal interest rates is identical to that in (27). Members of committee **(h)** weight their own opinions more heavily

[31] I suggest that if career concerns are present amongst members of the BoEMPC, and manifest themselves in much the same way as described by Lawrence Roos for members of the FOMC, it is not entirely incredulous to suggest that insiders will weight the opinions of insiders more heavily than outsiders. This fosters the prediction that insiders will be on the winning side of MPC decisions than outsiders, which is seen in practice. It would thus account for why Edmonds (1999) was able to observe that "the Bank representatives [*insiders*] have formed into a cohesive homogenous group".(*p*.12)

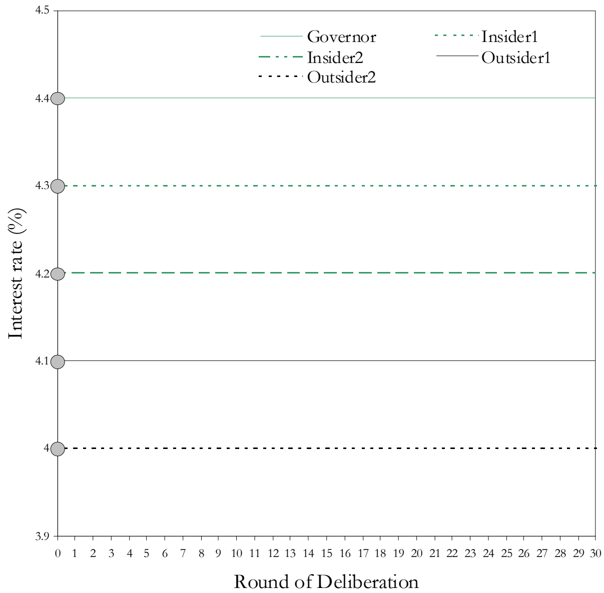
(A)

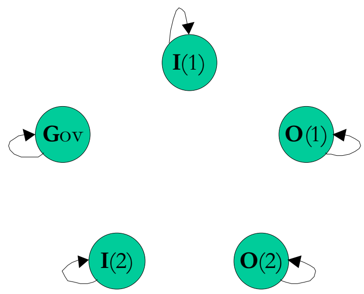




$$\mathbf{P} = \begin{bmatrix} 0.8 & 0.05 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.8 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.8 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.8 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.8 \end{bmatrix}$$

(B)

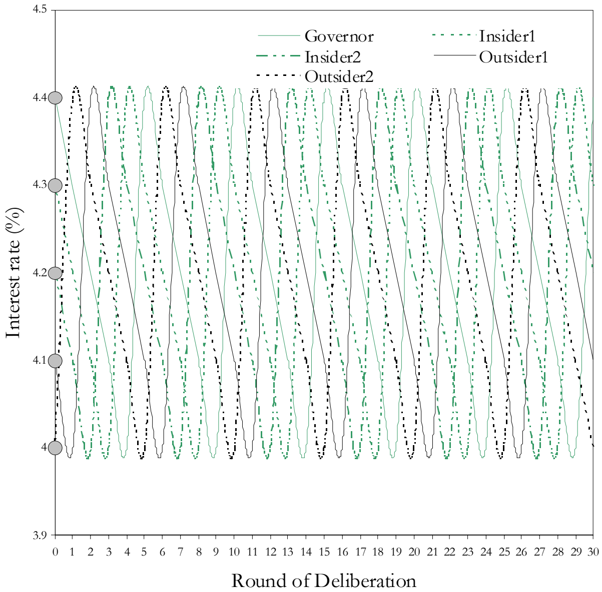


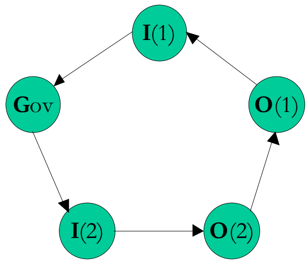


$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Panel 4.1:
Interest rate convergence paths for a 5 member MPC

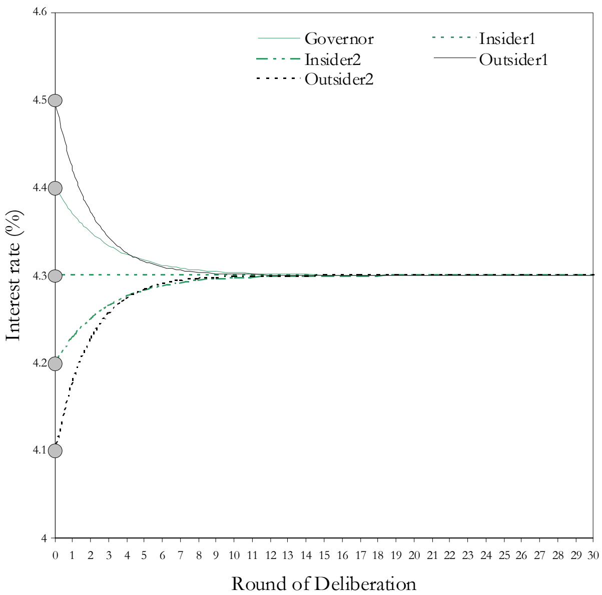
(C)

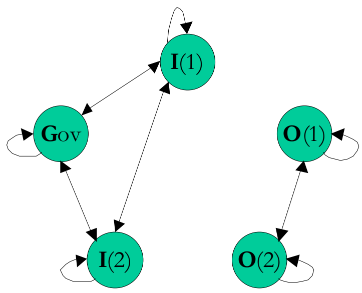




$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(D)

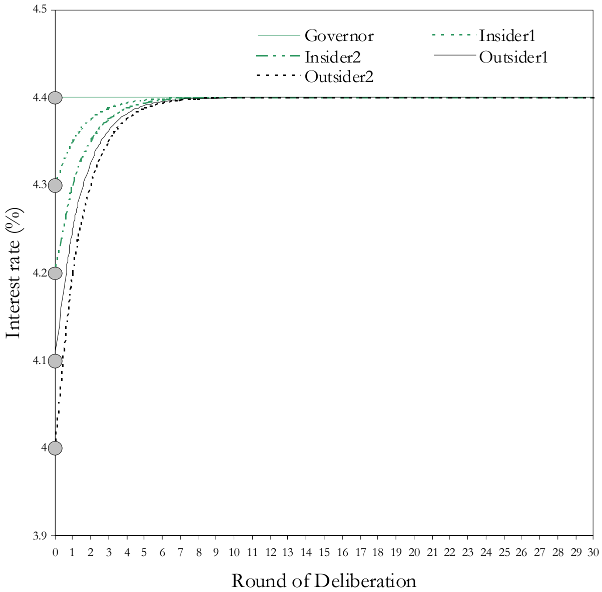


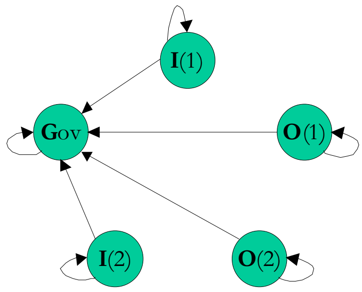


$$\mathbf{P} = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix}$$

Panel 4.2.
Interest rate convergence paths for a 5 member MPC (continued)

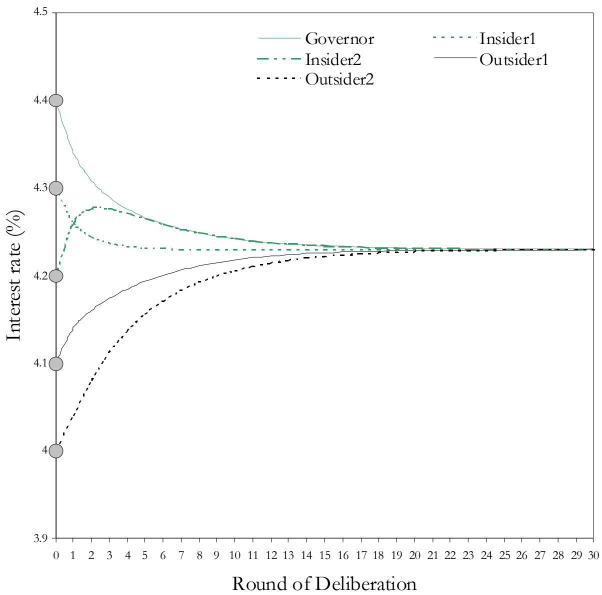
(E)

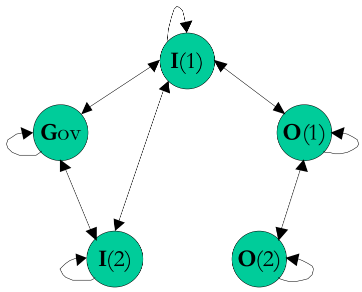




$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

(F)

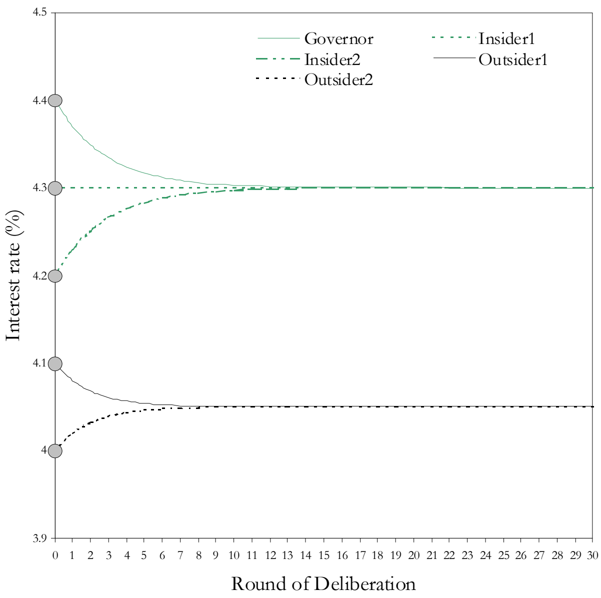


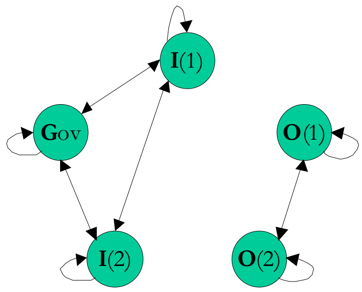


$$\mathbf{P} = \begin{bmatrix} 0.6 & 0.2 & 0.2 & 0 & 0 \\ 0.1 & 0.6 & 0.1 & 0.2 & 0 \\ 0.2 & 0.2 & 0.6 & 0 & 0 \\ 0.2 & 0 & 0 & 0.6 & 0.2 \\ 0 & 0 & 0 & 0.4 & 0.6 \end{bmatrix}$$

Panel 4.3:
Interest rate convergence paths for a 5 member MPC (continued)

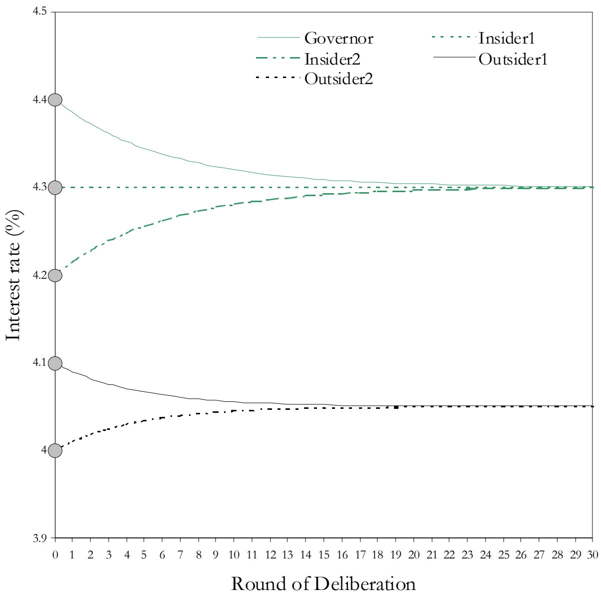
(G)

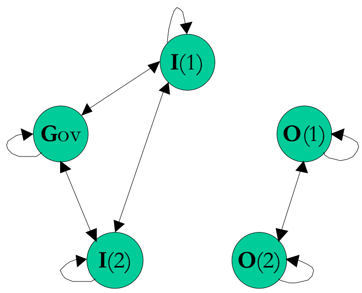




$$\mathbf{P} = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix}$$

(H)





$$\mathbf{P} = \begin{bmatrix} 0.9 & 0.05 & 0.05 & 0 & 0 \\ 0.05 & 0.9 & 0.05 & 0 & 0 \\ 0.05 & 0.05 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix}$$

Panel 4.4.
interest-rate Convergence Paths for a 5 member committee of Pragmatists and Egoists

than members of committee **(g)**. We have that

$$\mathsf{P}^{\text{Pragmatists}} = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 & 0 \\ 0.01 & 0.8 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix} \begin{matrix} \text{MEMBER} \\ \text{Governor} \\ \text{Insider 1} \\ \text{Insider 2} \\ \text{Outsider 1} \\ \text{Outsider 2} \end{matrix} \tag{29}$$

and

$$\mathsf{P}^{\text{Egoists}} = \begin{bmatrix} 0.9 & 0.05 & 0.05 & 0 & 0 \\ 0.05 & 0.9 & 0.05 & 0 & 0 \\ 0.05 & 0.05 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix} \begin{matrix} \text{MEMBER} \\ \text{Governor} \\ \text{Insider 1} \\ \text{Insider 2} \\ \text{Outsider 1} \\ \text{Outsider 2} \end{matrix} \tag{30}$$

where $\mathsf{P}^{\text{Pragmatists}}$ and $\mathsf{P}^{\text{Egoists}}$ correspond to the matrices in simulations **(g)** and **(h)** respectively. Compared to $\mathsf{P}^{\text{Pragmatists}}$, the influence matrix $\mathsf{P}^{\text{Egoists}}$ is notable in as far as *all* MPC members weight their own opinions more heavily, at the cost of the opinions of their peers. Members are less open to the opinions of others, so to speak: less *pragmatic*, and more *egoistic*. Because egoists by definition weight their own opinions more heavily than pragmatists, reaching a majority consensus takes *longer*. In the examples in PANEL 4.4, both committees still choose the same interest-rate, albeit it takes longer for members in **(h)** to arrive at a decision. This is borne out by eigenvalue analysis. Firstly, because the influence matrix is characterised by two disjoint communicating classes, both matrices have *two* eigenvalues with moduli equal to 1. In **(g)** and **(h)** it follows that

$$\delta(\mathsf{P}^{\text{Pragmatists}}_{Insiders}) < \delta(\mathsf{P}^{\text{Egoists}}_{Insiders}) \tag{31}$$

and

$$\delta(\mathsf{P}^{\text{Pragmatists}}_{Outsiders}) < \delta(\mathsf{P}^{\text{Egoists}}_{Outsiders}) \tag{32}$$

Egoistic *insiders* reach a consensus amongst themselves after $\xi = \frac{\log 0.008}{\log 0.85} \approx 30$ deliberative rounds, whereas pragmatists do so after $\xi = \frac{\log 0.008}{\log 0.7} \approx 14$ rounds. The same applies to outsiders, for whom members of the egoistic variety reach a consensus after $\xi = \frac{\log 0.008}{\log 0.8} \approx 22$ rounds, in contrast to $\xi = \frac{\log 0.008}{\log 0.6} \approx 9$ rounds for pragmatists.

8 Conclusion

This paper has adopted a bounded-rational approach to decision making by monetary policy committees. It has sought to account for the deliberation process by demonstrating how MPC members' views align when interest rate preferences are initially diverse. If members share initially diverse views, communication amongst MPC members is crucial in forging consensus. I showed how consensus may be achieved under a variety of different assumptions, and provided a formal mechanism which potentially explains how *autocratically collegiate*, *genuinely collegiate* and *individualistic committees* are able to reach a decision. The model potentially explains the process by which members of an MPC are persuaded to budge from their initial interest rate positions. In practice, MPCs *do* reach an agreement, which suggests that members do listen to each other and modify their beliefs in the course of deliberations. I suggest that this may be due to career concerns, members' subjective assessments of other's information, and the internal culture of an MPC.

An overriding conclusion which emerges from the analysis is that it is possible to populate MPCs with people who hold very different views about the economy and *still* reach an agreement. Secondly, and perhaps more obviously, MPCs should be populated by people who are willing to listen to the opinions of others. A consequence of not adhering to this recommendation is that reaching an agreement may not be achievable, as was demonstrated by Proposition 4 - when no member listens to the views of other members, consensus is not reached, unless one relaxes the assumption of initially diverse interest rate preferences. Yet the fact that what drives people to listen to one another is motivated by (i) career concerns and (ii) members' subjective assessments of others information raises important questions. Is, for example, the consensus outcome the best for the economy, especially given that the propensity for members to reach agreement is emphatically *not* driven by a desire to adopt the 'optimal' policy, but selfish motives such as career advancement?

The model may potentially explain Alan Blinder's (1998) contention that monetary policy committees reach decisions which are 'inertial' and 'regress towards the mean'. For instance, consider how *genuinely collegiate* MPCs reach a 'compromise' interest rate characterised by an average of initial beliefs weighted by the unique stationary distribution of the *influence matrix* as $\lim_{n\to\infty}\mathsf{P}^n$. The model raises further questions about the benefits of delegating monetary policy to a committee. It is often assumed that such an institutional arrangement is beneficial because members are able to pool their information. However, this precludes the fact that (i) any given member's assessment of others' information may be purely subjective -and wide off the mark - and (ii) agents may be boundedly rational, using a simple heuristic to update beliefs, as opposed to more robust and computationally burdensome procedures. Very few models of monetary policy decision making consider these possibilities. In this sense there exists considerable scope for this paper to be extended, especially if one relaxes the assumption of initially diverse interest rate preferences. This would entail that for example, some members may share identical beliefs prior to the deliberation process. The analysis in this paper has not considered this more general case. Secondly, one might compare the extent to which decision outcomes are different when members are *fully* as opposed to *boundedly* rational.

9 Appendix

9.A Proof of Proposition 1

To prove that a unanimous consensus will be reached by the committee if the influence matrix P is irreducible I draw heavily on Kemeny and Snell (1960) and Theil (1972). To establish the proof it is first necessary to draw on the following lemma:

9.A.1 Lemma 1

Let the $m \times m$ transition matrix P contain strictly positive elements $\mathsf{p}_{i,j}$ such that $\mathsf{p}_{i,j} \in [0,1]$ and $\Sigma_{i=1}^{m} = 1$. Denote ε as the smallest element of P by x, an $m \times 1$ column vector and Px the vector arising from pre-multiplying x by P. Denote by m_0 and M_0 the smallest and largest elements of x respectively, and let m_1 and M_1 denote the corresponding values for vector Px. Lemma 1 asserts that

$$M_1 \leqq M_0 \tag{33}$$

$$m_1 \geqq m_0 \tag{34}$$

and

$$M_1 - m_1 \leqq (1 - 2\varepsilon)(M_0 - m_0) \tag{35}$$

Proof of $M_1 \leqq M_0$: To show (33) and (35), begin by denoting x^* as an $m \times 1$ column vector obtained from x through replacing all but one element m_0 with M_0. Where there is more than one m_0 element in x assume that all but one of them are replaced by M_0. Now consider that we have that

$$\mathsf{x} = \begin{bmatrix} m_0 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n-1} \\ M_0 \end{bmatrix} \text{ and } \mathsf{x}^* = \begin{bmatrix} m_0 \\ M_0 \\ \cdot \\ \cdot \\ \cdot \\ M_0 \\ M_0 \end{bmatrix} \tag{36}$$

where $m_0 \leqq x_1 \leqq x_2, ...x_{n-1} \leqq M_0$. Let

$$\mathsf{x} \leqq \mathsf{x}^* \tag{37}$$

by assuming that each element of x^* is greater than its corresponding element in x by a nonnegative amount. Using the relationships outlined above, and because $\sum_{j=1}^{m} p_{i,j} = 1$, the i^{th} element of vector Px^* is captured by

$$\begin{aligned} p_{i,1}m_0 + \sum_{j=2}^{m} p_{i,j}M_0 &= p_{i,1}m_0 + (1 - p_{i,1})M_0 \\ &= M_0 - p_{i,1}(M_0 - m_0) \end{aligned} \tag{38}$$

Given that ε is the smallest element of the transition matrix P, it follows that it must be less than or equal to any element $p_{i,j}$, namely

$$\varepsilon \leqq p_{i,j} \tag{39}$$

Therefore we have that $\varepsilon \leqq p_{i,1}$. This implies that for vector Px^*,

$$M_0 - \varepsilon(M_0 - m_0) \geqq M_0 - p_{i,1}(M_0 - m_0) \tag{40}$$

As it has already been defined that $\mathsf{x} \leqq \mathsf{x}^*$, it follows that

$$M_0 - p_{i,1}(M_0 - m_0) \geqq M_1 \tag{41}$$

Using (40) and (41) it is possible to write

$$M_0 - \varepsilon(M_0 - m_0) \geqq M_1 \tag{42}$$

Assuming $M_0 \geqq m_0$ and $\varepsilon \in [0,1]$ in (42) establishes the first part of (33), namely $M_1 \leqq M_0$.

Proof of $m_1 \geqq m_0$: Proving (34) draws on the result shown in (33), namely (42). First consider the difference between the i^{th} element in the $(m \times 1)$ column vectors Px^* and Px, where x_j corresponds to the j^{th} element of x and x_j^* denotes the equivalent element of x^*. Specifically, write that

$$\sum_{j=1}^{m} p_{i,j}(x_j - x_j^*) \tag{43}$$

Given the assumption of $\mathsf{x} \leqq \mathsf{x}^*$ coupled with $\mathsf{p}_{i,j} \in [0,1]$ it follows that (43) cannot be positive. Now supplant x with $\mathsf{-x}$ such that Px now becomes $\mathsf{P(-x)} = \mathsf{-Px}$. This implies that the smallest and largest elements of $\mathsf{-Px}$ are $-M_1$ and $-m_1$ respectively. Therefore re-write (42) as

$$-m_0 - \varepsilon(m_0 + M_0) \geqq -m_1 \tag{44}$$

The inequality (44) establishes that $m_1 \geqq m_0$. *QED.*

Proof of $M_1 - m_1 \leqq (1-2\varepsilon)(M_0 - m_0)$: To demonstrate that (35) holds hinges on the key results used to prove (33) and (34). Summing (42) and (44) yields

$$M_1 - m_1 \leqq (1 - 2\varepsilon)(M_0 - m_0) \tag{45}$$

which confirms (35). *QED.*

9.A.2 Convergence of a Regular Markov Chain to a Stationary Distribution

Using Lemma 1 it is now proved that consensus is reached by all MPC Members when the opinion matrix is regular. In keeping with the previous section the elements of the opinion matrix are assumed to have the values $\mathsf{p}_{i,j} \in [0,1]$ where $\Sigma_{i=1}^{m} \mathsf{p}_i = 1$, $i = (1, 2, ...m)$. Specifically, the proof requires demonstration that for any regular matrix P^t, $\lim_{t\to\infty} \mathsf{P}^t = \boldsymbol{\iota}\boldsymbol{\pi}'$, where is a $m \times 1$ vector of ones, and $\boldsymbol{\pi}'$ is a $1 \times m$ vector of probabilities. In other words, in the limit as $t \to \infty$ P^t converges on an $m \times m$ matrix with identical rows, entailing that elements in each column are identical. Write this as

$$\lim_{t\to\infty} \mathsf{P}^t = \begin{bmatrix} \pi_1 & \pi_2 & \cdot & \cdot & \cdot & \pi_{m-1} & \pi_m \\ \pi_1 & \pi_2 & \cdot & \cdot & \cdot & \pi_{m-1} & \pi_m \\ \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & & & & \cdot & \cdot \\ \pi_1 & \pi_2 & \cdot & \cdot & \cdot & \pi_{m-1} & \pi_m \\ \pi_1 & \pi_2 & \cdot & \cdot & \cdot & \pi_{m-1} & \pi_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\pi}' \\ \boldsymbol{\pi}' \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\pi}' \\ \boldsymbol{\pi}' \end{bmatrix} = \boldsymbol{\iota}\boldsymbol{\pi}' \tag{46}$$

where $\Sigma_{i=1}^{m}\pi_i = 1$, $i = (1, 2, ...m)$. Now introduce the $m \times 1$ vector $\mathbf{i}_j$. All elements of are assumed to be zero apart from the j^{th} element, which equals 1. Further, consider the matrix $\mathbf{A}$ which has m columns. Post-multiplication of $\mathbf{A}$ by $\mathbf{i}_j$ yields an $m \times 1$ vector identical to the j^{th} column of $\mathbf{A}$.

Now envisage the chain of vectors $\mathbf{i}_j, \mathbf{P}\mathbf{i}_j, \mathbf{P}^2\mathbf{i}_j, \mathbf{P}^3\mathbf{i}_j, ..., \mathbf{P}^t\mathbf{i}_j, ...$denoting the largest corresponding elements $M_0, M_1, M_2, M_3, ..., M_t, ...$ and the smallest elements $m_0, m_1, m_2, m_3, ..., m_t, ...$ Applying Lemma 1, and using the relationship $\mathbf{P}^t\mathbf{i}_j = \mathbf{P}(\mathbf{P}^{t-1}\mathbf{i}_j)$ it is possible to conclude that

$$M_0 \geqq M_1 \geqq M_2 \geqq M_3 ... \geqq M_t, \tag{47}$$

$$m_0 \leqq m_1 \leqq m_2 \leqq m_3 \leqq ... \leqq m_t \tag{48}$$

and

$$M_t - m_t \leqq (1 - 2\varepsilon)(M_{t-1} - m_{t-1}) \tag{49}$$

Substituting $M_{t-1} - m_{t-1}$ with $(1 - 2\varepsilon)(M_{t-2} - m_{t-2})$ gives

$$M_t - m_t \leqq (1 - 2\varepsilon)^2(M_{t-2} - m_{t-2}) \tag{50}$$

Systematically repeating this procedure for $(M_{t-2} - m_{t-2}), (M_{t-3} - m_{t-3})$ through to $(M_{t-(t-1)} - m_{t-(t-1)})$ - in other words using backwards substitution - gives

$$d_t \leqq (1 - 2\varepsilon)^t d_0 = (1 - 2\varepsilon)^t \tag{51}$$

where $d_t = M_t - m_t$ and $d_0 = 1$, the latter term being the value of the largest element of $\mathbf{i}_j$, $M_0 = 1$, less the smallest element, $m_0 = 0$. As $(1 - 2\varepsilon) \in [-1, 1]$, d_t necessarily converges to zero as $t \to \infty$: essentially, as M_t and m_t are the largest and smallest elements in the j^{th} column of $\mathbf{P}$, all elements of the column converge to the same value, namely π_j in (46). *QED.*

Proof that π_j is positive: It has previously been shown that $m_1 \leqq m_t$ [eqn. (48)]. As m_1 is identical to the smallest transition probability in row j of element of matrix $\mathbf{P}$, namely $p_{1,j}, p_{2,j}, ..., p_{m-1,j}, p_{m,j}$, and each $p_{i,j} \geqq \varepsilon > 0$, π_j must be positive. *QED.*

9.B Proof of Proposition 2

To prove Proposition 2 it is first necessary to draw on the theory of *absorbing chains*. For any $m \times m$ transition matrix P, a state s_i is defined as *absorbing* if and only if $p_{jj} = 1\ \forall j = 1, 2, ..., m$. In the context of the model, an *absorbing state* is characterised by an MPC member listening *only* to himself, thereby placing no weight on the opinions of his colleagues. Should absorbing states arise, it is possible to permutate and subsequently partition the rows and columns of the corresponding influence matrix into the block form

$$\mathsf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{B} & \mathbf{C} \end{bmatrix} \tag{52}$$

where $\mathbf{I}$ is a $(m - s) \times (m - s)$ identity matrix, $\mathbf{0}$ is $(m - s) \times s$ array of zeros and $\mathbf{B}$ and $\mathbf{C}$ are $s \times (m - s)$ and $s \times s$ arrays of transient states respectively. This structure of (52) implies that members belonging to the initial $(m - s)$ rows weight only their own opinions, whereas individuals pertaining to the last s rows with are influenced by the opinions of members who weight only their opinions in the first $(m-s)$ rows. It follows from (52) that

$$\mathsf{P}^n = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \sum_{i=0}^{n-1} \mathbf{C}^i\mathbf{B} & \mathbf{C}^n \end{bmatrix} \tag{53}$$

where by **Theorem 1.11.1** of Kemeny and Snell (1960), $\lim_{n\to\infty} \mathbf{C}^n = 0$. Using these theorems, proof of Proposition 2 now follows:

Proof of Proposition 2: For an m member committee if any member j is influenced only by himself, and influences all other members, either directly or indirectly a *unanimous consensus* will be reached by the committee. By (52) and **Theorem 1.11.1** of Kemeny and Snell, members' beliefs will necessarily converge to those of member j. Treating $\mathbf{I}$ as a (1×1) matrix whose only element is $p_{11} = 1$, it follows that in the limit all elements in the remaining $(m-1)$ rows of the first column of the influence matrix will converge to a single column of ones, with all other elements converging to zero.

$$\lim_{n\to\infty} \mathsf{P}^n = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \tag{54}$$

QED.

9.C Proof of Proposition 3

To prove Proposition 3 I draw on results from the proof of Proposition 1. For an m member committee, begin by introducing the block matrix

$$\mathsf{P} = \begin{bmatrix} \boldsymbol{\Upsilon} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Delta} \end{bmatrix} \tag{55}$$

where Υ and Δ comprise two disjoint communicating classes. Specifically, let $\boldsymbol{\Upsilon}$ denote a $(m-s)\times(m-s)$ bloc of opinion weights for insiders and Δ denote a corresponding $s\times s$ bloc for outsiders. In the limit it necessarily holds that

$$\lim_{n\to\infty} \mathsf{P} = \begin{bmatrix} \boldsymbol{\Upsilon}^\infty & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Delta}^\infty \end{bmatrix} \tag{56}$$

Given (56) it follows that each bloc can be treated as a matrix in its own right (as the elements of Υ and Δ do not influence each other). Because both matrices each comprise a *single aperiodic recurrent class* (i.e. all members within each group are strongly connected) the results for Proposition 1 apply to the $(m-s)\times(m-s)$ matrix Υ and the $s\times s$ matrix Δ. Thus the limiting distribution of P takes the form.

$$\lim_{t\to\infty} \mathsf{P}^t = \begin{bmatrix} \pi_1 & \cdot & \cdot & \cdot & \pi_{m-s} & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & & & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & & & \cdot & \cdot \\ \pi_1 & \cdot & \cdot & \cdot & \pi_{m-s} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & \pi_{m-s+1} & \cdot & \cdot & \cdot & \pi_m \\ \cdot & \cdot & & & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & \pi_{m-1} & \cdot & \cdot & \cdot & \pi_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\pi}'_{Insiders} \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\pi}'_{Insiders} \\ \boldsymbol{\pi}'_{Outsiders} \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\pi}'_{Outsiders} \end{bmatrix} \tag{57}$$

The first $(m-s)$ rows will converge to a stationary distribution characterised by the first $(m-s)$ columns containing strictly positive elements and the latter s columns comprising zeros. The last s rows will converge to a limit characterised by elements in the first $(m-s)$ columns comprising zeros, and the remaining s columns containing strictly positive elements. Assuming $(m-s) > s$ implies that insiders reach a *majority consensus*. In the presence of a tie (i.e. when m is not odd), assume that the chairman has a casting vote. *QED*.

The special case of a *unanimous consensus* being reached in the presence of *two aperiodic recurrent* classes is characterised by both groups converging on the same interest rate. Again, assuming an m member committee with initially diverse preferences, this occurs when

$$\sum_{j=1}^{m-s} \pi_j \mathsf{i}_{j,0} = \sum_{j=m-s+1}^{m} \pi_j \mathsf{i}_{j,0} \tag{58}$$

where π_j denotes elements of matrix (18) in the jth column and the $\mathsf{i}_{j,0}$s correspond to the elements of the period $t=0$ *belief* vector $\mathsf{I}^{[0]}$ as in (4). The first $m-s$ elements correspond to insiders' initial beliefs, and the latter s elements correspond to the initial beliefs of outsiders. The transpose of the vector is given by

$$\mathsf{I}^{[0]\prime} = [\overbrace{\mathsf{i}_{1,0}, ..., \mathsf{i}_{m-s,0}}^{m-s \text{ insiders}}, \overbrace{\mathsf{i}_{m-s+1,0}, ..., \mathsf{i}_{m,0}}^{s \text{ outsiders}}] \tag{59}$$

An equivalent representation of (4) is given by

$$\boldsymbol{\pi}'_{Insiders}\mathsf{I}^{[0]} = \boldsymbol{\pi}'_{Outsiders}\mathsf{I}^{[0]} \tag{60}$$

where $\boldsymbol{\pi}'_{Insiders}$ is the vector capturing the unique stationary distribution for insiders, and $\boldsymbol{\pi}'_{Outsiders}$ is the corresponding vector for outsiders.

9.D Proof of Proposition 4

If members are neither directly nor indirectly influenced by their colleagues, they effectively place weight 1 on their own opinions. The influence matrix is thus an $m \times m$ identity matrix. This implies that

$$P = P^n, \qquad \forall\, n,\ n = 1, 2.....\infty \tag{61}$$

If $P = P^n$, then

$$\mathsf{I}^{[0]} = \mathsf{I}^{[n]}, \qquad \forall\, n,\ n = 1, 2.....\infty \tag{62}$$

as from (5) and (7) we have that $\mathsf{I}^{[n]} = \mathsf{P}^n\mathsf{I}^{[0]} = \mathsf{P}\mathsf{I}^{[0]} = \mathsf{I}^{[0]}\ \forall\ n$. Initial beliefs remain unchanged irrespective of the stage of the deliberation process, and no consensus is achieved. *QED.*

9.E Proof of Proposition 5

By **Theorem XV.6.1** of Feller (1968), all states of an irreducible chain are of the same type. Thus for a periodic chain, if there are m recurrent states then P is cyclic of order m. This implies that member j is only influenced by any another member k every $t = m$ intervals. In all other $t - 1$ periods j is not influenced by k.

Consider, for instance, a three member MPC sitting in a circle, where each member only weights the opinion of the person sitting to the right of him. Let the influence matrix P and subsequent powers to which it is raised be given by

$$\mathsf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \mathsf{P}^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \mathsf{P}^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \mathsf{P}^4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{63}$$

The matrix P is of period 3. Not only does $\mathsf{P} = \mathsf{P}^4$, but more generally $\mathsf{P} = \mathsf{P}^{n3+1}$, $\forall\ n = 1, 2, ...\infty$. Given that $\mathsf{P} \neq \mathsf{P}^2 \neq \mathsf{P}^3$ and $(\mathsf{P} = \mathsf{P}^{n3+1}) \neq (\mathsf{P}^2 = \mathsf{P}^{((n+1)3)+1}) \neq (\mathsf{P}^3 = \mathsf{P}^{((n+2)3)+1})$, $\forall\ , n = 1, 2, ...\infty$, P^n can *never* converge. Extending this to the case of m members (and thus m cycles) it therefore follows that

$$\mathsf{P} \neq \mathsf{P}^2 \neq ..... \neq \mathsf{P}^m \tag{64}$$

and

$$(\mathsf{P} = \mathsf{P}^{nm+1}) \neq (\mathsf{P}^2 = \mathsf{P}^{((n+1)3)+1}) \neq ... \neq (\mathsf{P}^{\mathsf{m}} = \mathsf{P}^{((n+2)m)+1}) \tag{65}$$

Now apply (65) to the iteration process given by (7). The beliefs which emerge following subsequent revisions have the following properties:

$$\begin{aligned} \mathsf{I}^{[0]} &= \mathsf{I}^{[nm]} \\ \mathsf{I}^{[1]} &= \mathsf{I}^{[nm+1]} \\ &. \\ &. \\ &. \\ \mathsf{I}^{[m-1]} &= \mathsf{I}^{[nm+(m-1)]} \end{aligned} \tag{66}$$

for all $n = 1, 2, ...\infty$. As P^n cannot converge, neither do initial beliefs. Rather, revised estimates at each stage of the deliberation process cycle in tandem with the period m of the listening matrix such that member j is only influenced by any another member k every m^{th} interval.[32] *QED.*

It is noted that the results for Proposition 5 do not extend to the case where the influence matrix P is characterised by m sets of *cyclic recurrent classes.* I stated earlier that it is possible

[32] See Feller (1968) for an an equivalent proof for the generalization to m states.

for (i) unanimous consensus to be reached when there to exist some linear combination of opinion weights and initial beliefs such that $\mathsf{I}^{[1]} = \mathsf{I}^{[2]} = ... = \mathsf{I}^{[n]}$ or (ii) a majority consensus arising when members beliefs converge on the same interest rate for $\mathsf{I}^{[n]}$, $n = 1, 2, ...\infty$, such that the *composition* of the majority switches regularly in sync with the period of the cycle. Illustrative examples of such behaviour are henceforth given.

9.E.1 Case (i): Unanimous Consensus

Consider a nine member committee characterised by the following belief matrix P:

$$\mathsf{P} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{67}$$

which is representable by the following bloc form, where each bloc is a 3×3 sub-matrix:

$$\mathsf{P} = \begin{bmatrix} 0 & A & 0 \\ 0 & 0 & B \\ C & 0 & 0 \end{bmatrix} \tag{68}$$

The dynamic behaviour of (68) is clearly analogous to that given by (65). Now introduce a belief vector $\mathsf{I}^{[0]}$, the transpose of which is given by

$$\mathsf{I}^{[0]} = [4.5, 4.4, 4.3, 5.0, 4.6, 3.6, 4.9, 4.2] \tag{69}$$

Given the influence structure in (65) and its corresponding cycling behaviour, it can be shown that revised estimates in *every* period converge to an interest rate of 4.4%, namely

$$\mathsf{I}^{[n]} = [4.4, 4.4, 4.4, 4.4, 4.4, 4.4, 4.4, 4.4], \qquad n = 1, 2, ...\infty. \tag{70}$$

9.E.2 Case (ii): Majority Consensus

Under *majority consensus*, the *composition* of the majority switches regularly in sync with the period of the cycle. Again, consider the influence matrix given by (67) and its corresponding bloc from in (68). If a belief vector $\mathsf{I}^{[0]}$ is introduced such that

$$\mathsf{I}^{[0]\prime} = [4.5, 4.4, 4.3, 5.0, 4.0, 3.6, 4.9, 4.2], \tag{71}$$

it follows that revised beliefs using the iterative procedure in (7) in successive rounds are given by

$$\mathsf{I}^{[1]} = \begin{bmatrix} 4.2 \\ 4.2 \\ 4.2 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \end{bmatrix}, \mathsf{I}^{[2]} = \begin{bmatrix} 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.2 \\ 4.2 \\ 4.2 \end{bmatrix}, \mathsf{I}^{[3]} = \begin{bmatrix} 4.4 \\ 4.4 \\ 4.4 \\ 4.2 \\ 4.2 \\ 4.2 \\ 4.4 \\ 4.4 \\ 4.4 \end{bmatrix}, \mathsf{I}^{[4]} = \begin{bmatrix} 4.2 \\ 4.2 \\ 4.2 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \\ 4.4 \end{bmatrix} \tag{72}$$

Clearly, a majority consensus is reached at every iteration with six out of nine members preferring to settle for a rate of 4.4%. However, as stated above, the composition of the majority switches cyclically from period to period. More formally we have that $\mathsf{I}^{[n]} = \mathsf{I}^{[n+\nu m]}$where n denotes the deliberative round, m the period and $\nu = 1, 2, ...\infty$. This implies that all attempts by the chairman decide to continue deliberating in the hope of increasing the size of the majority will have no effect. Re-expressing $\mathsf{I}^{[0]}$in the bloc form

$$\mathsf{I}^{[0]} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{73}$$

where X, Y and Z denote (3×1) sub-matrices of the first, second and last three elements, a sufficient condition for *majority consensus* to be reached each period is thus given by

$$\begin{aligned} BX &= CX, \\ AZ &= BX, \\ AX &= CZ \end{aligned} \tag{74}$$

and

$$\begin{aligned} BZ &= BX, \\ CX &= CZ, \\ AZ &= AX \end{aligned} \tag{75}$$

Here, A, B and C denote the blocs in (68) and the elements in each sub-matrix in each multiplication are assumed to converge to the same value.

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